

## FRACTURE MECHANICS APPROACHES IN THE DESIGN OF AEROSPACE VEHICLES

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Abstract

Damage tolerance requirements, which are considered in aeronautical military specifications and in spacecraft design criteria, are now entering also in civil regulations.

Adequate methodologies are to be employed to fulfill such requirements. They interest the loading condition analysis, the dynamic response of the structure, the defect detection by means of NDI methods, the material qualification as concern residual strength and crack propagation in ductile metal behaviour and a comprehensive safety philosophy.

With, the aim of contributing to the knowledges in such fields a joint research effort is carried out in Italy by Universities and Industries under the sponsorship of National Research Council.

Among the objectives of this research, this paper summarizes some results obtained on: 3-dimensional stress intensity factor; models for ductile fracture; crack propagation in riveted structure; crack growth computation; minimum weight design.

A focusing on NDI method with their capability evaluation and a description of fracture mechanics application in an industrial spacecraft design are also reported.

1. Foreword

The development of damage tolerant structure technology to ensure safety of aerospace structure promoted important research activities in fracture mechanics and in other concerned areas as the NDI methods.

The need of ensuring in Italy an adequate technological level in such a field suggested to the Italian "National Research Council" (C.N.R.) to support a research program, coordinated between University and Industry.

Beside this program the development of industrial activities, particularly international programs such as the SPACELAB, created other sources of knowledges and needs of applications in this area.

The scope of this paper is a presentation of research and development activities performed in Italy and their setting in the present state of art.

2. Safety and structural damage

The safety of modern aircraft structures largely depends on their capability to operate in the presence of damage without suffering catastrophic failures for a specified operational time. The structural damage may stem directly from the manufacturing processes or may arise during aircraft operations due to such causes as fatigue, corrosion and accidental damaging events.

The damage can grow under service loading and environment and reach dimensions which are felt dangerous, as the residual static strength decreases to such an extent that a catastrophic failure results reasonably probable.

Therefore if safety must be guaranteed, the structure must be designed in such a manner that, should damage occur, the structure would retain adequate

strength until the damage is detected in a scheduled inspection.

At first the design against structural damage was a challenge only for the designers of commercial transport airplanes. In fact airworthiness standards for this category of aircraft were demanding the consideration of in-flight damage in the early sixties.

The basic point of view of this airworthiness philosophy was, until recently, the identification of structural damage with fatigue cracks. Therefore the safety requirement was that any critical part of the airplane structure had either adequate fatigue strength (capability to withstand the repeated loads expected in service without crack occurrences) or adequate "fail-safe" strength (capability to withstand specified loads (°) after a fatigue failure of a single principal structural element).

These rules were the reference frame for the design of all existing commercial transport airplanes.

The manufactures of such airplanes have successfully dealt with the problem of the "fail-safe" structures through design practise based on the multiple load paths, crack stopping ability and slow crack growth. The existing structural criteria nevertheless do not exhaust all the important damage conditions and may conduct to unsafe design if not applied with great ingenuity. Service experience has in fact indicated that fatigue cracks may develop despite exhaustive analysis and testing; corrosion may weaken the structure and accidental damage may occur. Recognizing that the existing rules can conduct to an unsafe design, an action has been undertaken by FAA to amend and update existing fatigue and fail safe requirements. The new approach is based on the idea to utilize, where practical, structural design based on damage tolerant and to utilize the more conventional fatigue (safe-life) design only for that particular structures which are impractical for a damage tolerance design. The damage tolerance criteria which will guide the design of new generations of transport airplanes can be summarized in the statement that "the strength, detail design, and fabrication of the airframe structure must be such as to avoid catastrophic failure due to fatigue, corrosion, or accidental damage throughout its operational life".

In the field of military aircraft the conventional safe-life concept was considered until recent time as the best approach for achieving structural safety and durability. The goal of this approach lies in designing a fatigue crack free structure for its service life. To reach such a goal it is felt adequate to use full-scale-fatigue results and a scatter factor to guarantee the structural safety of all the fleet. Service experience has nevertheless indicated that in several cases this approach did not assure the required safety. In particular an initial flaw in a critical area could cause early operational failures even though the test article demonstrated the required durability throughout in fatigue testing.

In other words the assumption, typical of the safe life design of an initially unflawed structure

(°) Such loads are a given percentage of the limit load.

may lead to an unsafe design for those airplanes of the fleet which contain flaws larger than the ones of the test article.

The realization of this state of thing promoted a significant review of the philosophy for achieving structural safety in military aircraft. A leading action in this area came from the U.S. Air Force since the 1969-70 time period. This action is now reflected in a specification issued in 1974, (MIL-A-83444) which establishes the new requirements for the safety of flight structures.

In the current Air Force approach the safe-life design has been abandoned. Instead structural safety is obtained by requiring damage-tolerant structures. Damage tolerance is intended as the capability of the structure to accommodate flaws, induced either in manufacturing or in service, through a slow crack growth or a fail-safe behaviour. Fail-safety is there intended as the capability of the structure to operate with a primary structural element failed for a specified period of time. Such a capability can be obtained either with multiple load paths or with crack arrest ability. Slow crack growth is the capability to operate with flaws or cracks which grow due to load/environment spectra without attaining critical dimensions in a specified period of time. Damage tolerance must be guaranteed in all the critical structural areas; such areas must be carefully identified by an appropriate selection logic.

Another field where damage-tolerance concepts are playing an important role is the design of new generation multimission aerospace vehicles.

Here material, structural configuration, and manufacturing are typical, and past experience cannot be easily transferred in the design of such components where welding of high strength light alloys (like 2219) is the current trend.

Damage tolerance design of aerospace vehicle is currently considered in NASA SP-8057 a document issued to assist in the preparation of structural requirements and specifications for Space Shuttle. This document can also be considered a reference frame for structural design for the Space-Lab, a fundamental payload of the Space Shuttle.

Two approaches are considered, namely safe crack growth life (safe-life) and fail-safe.

The first approach is conceptually equivalent to the slow crack growth approach of aircraft design and the second is coincident with the equal name concept of the aircraft design.

Damage tolerance requirements has prompted substantial innovations in the design procedures, through the the introduction and/or the reconsideration of disciplines in the design offices and test facilities.

In fact damage tolerant design procedures depend heavily on fracture mechanics for predictions of crack propagation rate and fracture strength and on NDI methodologies to assess allowable initial flaw dimensions.

Research and development in these areas have been steadily increasing in the recent period and currently a body of knowledge is available to face the main design problems with some confidence. However the soundness of a damage-tolerant design at present must be assessed against a body of data to be collected through complex and costly test programs. In particular tests must be conducted to verify the rate of growth under realistic load conditions and the residual strength for obvious or detectable flaws.

Further the reliability of detection procedures must be adequately substantiated.

There is therefore the need of further advance

in such fields to put the damage tolerant design technology on more efficient and reliable bases.

Such an advance could be obtained through the conduction of fundamental researches on the main topics of fracture mechanics and NDI procedures, as well as through the critical appraisal of significant damage tolerant designs.

### 3. Current status of Damage Tolerant design technology.

The design of damage tolerant structures can take up different features in the various fields of application (commercial or military aircraft, or aerospace vehicles) due to large differences in configurations, operational environment and usage.

Nevertheless a common base supports the design methodologies, which, essentially, are aimed at predicting and substantiating by tests the growth of a flaw or a fatigue crack.

Three basic tools are therefore necessary:

- NDI procedures to characterize the initial flaw dimensions;
- prediction methods and substantiation test procedures to evaluate the growth of a crack under realistic load conditions, and;
- prediction methods and substantiation test procedures to evaluate the residual strength of a cracked structure.

The current status of the knowledge on these topics and the areas of potential improvements can be better appreciated referring to a typical design case: the Module Structure of the SPACE-LAB.

SPACE-LAB is the European Space Laboratory that will be used over the next decades for scientific and technological space research, in conjunction with the United States Space Transportation System, the SPACE-SUTTLE.

The responsibility of the Module Structure elements has been assigned to AERITALIA that is one of the eleven Aerospace Companies joined in a Consortium lead by ERNO VFW-FOKKER.

#### 3.1 Design philosophy

The primary structure of the module (fig. 1) comprises the cylindrical shells and the cone end closures forming the pressurized compartment that provides the habitation environment and the structural support for the experimental equipment.

The philosophy of the design of such an advanced manned reusable vehicle was aiming at the development of a structure of minimum weight and reduced cost, capable of performing a defined number or repeated missions in a specified environment with a prescribed level of safety and reliability.

The development of SPACELAB has been essentially based on: - Analyses - Tests - Non Destructive Inspections.

The structural analysis utilizes the more advanced computational methods to provide the overall information on the stress field of the structure, and traditional hand calculations to provide the local stress values accounting for the local details of the structure design.

The analysis is usually performed in three steps: The Static verification to evaluate the strength and rigidity of the structure under the extreme values of the design static loads.

The Fatigue verification to evaluate the behaviour of the major structural elements under the repeated loads.

The Fracture Mechanics verification to assess the



behaviour of the structure in presence of defects.

o The structural tests are a fundamental support to the analysis and have become an essential part of the design.

Usually the structural tests cover:

- Tests on the basic material performed to evaluate in a consistent way the material structural behaviour (static strength, fatigue properties, fracture mechanics properties).
- Tests on components, performed on specified selected components of complex design or critical behaviour performed to substantiate the analysis.
- Test on structural elements subsystems as on the complete structure to verify the adequacy of the design in its totality by means of static and fatigue tests reproducing the loading conditions expected during the life.

o The Non Destructive Inspections provide the means to detect, in the various structural elements, defects of specified dimensions with an assigned probability of detection on which basis a detailed evaluation of the criticality of the presence of these defects and of their propagation under loads is performed.

### 3.2 Fatigue and Fracture Mechanics Verifications

SPACELAB Module structural elements have been designed for what concerns life duration according the following approaches:

- Safe life design based on conventional fatigue life. No consideration is given to potential undetected crack-like defects existing into the structure; the useful life of the structural elements subjected to respected load conditions is evaluated on the basis of material properties obtained from suitable tests such as the S-N curve, accounting for the number of cycles required to initiate a visible fatigue crack.

- Safe crack growth life based on Fracture Mechanics considerations. The structural elements are assumed to have defects of defined shapes and sizes; fracture mechanics analysis allows to evaluate the criticality of the element by defining the residual strength in relation to the load and environmental conditions existing in the operational conditions. The useful life is defined by analyses, supported by extensive testing to evaluate the material properties, as the period of time for an initial defect to grow under specified loads to a critical size.

The design approach is strongly dependent on the capability of the non destructive inspection methods to detect, with a given probability a crack or defect of specified size. The inspection criteria ensure that the defects do not grow within two inspection intervals up to a dimension that can endanger the safety of the element.

### 3.3. Evaluation of material properties

The selection of the aluminium alloy Al 2219 - T851 for the construction of the primary structure of the SPACELAB module, has been made accounting for its high strength properties associated with very good toughness characteristics. The welding properties of the alloy welded with the T.I.G. process with Al 2319 as filler are extremely good, a condition that has favored as well the selection.

As for the majority of metallic materials of common application in the aerospace constructions, the conventional static properties of Al 2219-T851 are well established; statistically based design allowable have been utilized directly in the design

with limited experimental investigations to validate the data available.

For what concerns the fatigue data, available information on the basic properties behaviour have been utilized. An investigation has been performed for AERITALIA by Istituto di Aeronautica Università di Pisa (44) on the fatigue behaviour of welded joints in presence of a offset ( $d/t = 0-25\%$ ). From the endurance data families of S-N-P curves have been obtained from which the influence of the offset has been evaluated through a fatigue stress concentration factor that resulted to be  $K_F = 1+2,4 d/t$ . These information have allowed to account in the analysis for rather common defect of a mismatch resulting in butt welding.

On the contrary Fracture Mechanics and Crack Propagation data needed for the safe crack growth verification were, as for the majority of the material, very poor and missing of the statistical information necessary to derive the design allowable data.

To overcome this lack of consolidated data, relative to the very material used in the construction of the Module, that is to say to account also for the influence of the various manufacturing processes applied, in "ad hoc" experimental test program has been performed at AERITALIA to generate, according to the specific design needs, the necessary data.

More general information on the material have been obtained from the research sponsored by CNR.

The Aeritalia test program of fracture mechanism (45) covers:

- a) Static Fracture on basic parent material and on welded material of different thickness representative of the shell skins and of the shell weldments.
- b) Cyclic Load Propagation Tests on the basic parent material and on welded material of the same thickness investigated in static tests.

In both cases through and part-through cracks were investigated.

The test program allowed the acquisition of the data necessary for Fracture Mechanics verifications. In particular the results of the static tests were summarized in  $K_t$ -thickness relationships both for parent and welded material, and the crack growth data were fitted through the Collipriest correlation formula.

### 3.4 Description of Analyses procedures and results

The final objective of the fatigue and fracture mechanics analysis of the pressurized structures, is the verification that the required life of specified missions is fulfilled.

A limited number of selected locations (about eighty) has been considered for analysis. Selection has been made considering the areas of high stresses, the areas of difficult or impossible inspection and the elements for which is required the leak before burst verification. The selected locations include external shell walls, weldments, frames, longerons, flanges, fittings and joining elements.

The investigations performed are aimed at estimating the probable size of original flaws and cracks, tracking the sources of these defects and establish appropriate controls, assessing the impact of initial size of defects on the flight worthiness, and determining the optimum inspection methods and frequencies relevant to each component selected for fracture control.

The analysis procedures are based on initial crack lengths, linear elastic fracture mechanics,

sequences of events, material properties and stress spectra.

The evaluation of the specific fracture behaviour for the components safelife, fail safe as well as "leak before break" properties are established and the inspection procedures are selected which will prevent catastrophic damage to the fracture critical parts.

A detailed analysis of the stress field in the areas considered for verification has been performed starting from the overall information obtained from the general programs.

The fatigue analysis has been performed using a standard program based on the linear damage accumulation theory.

The computations of the crack growth has been made using a special program that, starting from the type and initial dimensions of the defect, local spectra and material properties, gives the growth of the cracks as a function of the number of missions (fig. 2).

For each location starting from an initial defect size it has been evaluated the size for the part through defect to become through and the size for through defects to become critical and the number of mission associated is determined and compared with required life ( $50 \times 4 = 200$  missions).

After minor redesign of some details in few areas, the final verification has confirmed that no element is critical for fatigue, being the maximum damage equal to 0.08 largely inferior to the requested one ( $D = 0.25$ ); the fracture mechanics analysis has confirmed as well, that there are no critical areas in the Module Structure the element with the shorter life surviving up to 350 missions.

### 3.5. Plans for Verification Tests

In the frame of the qualification program of the SPACELAB Module Structure, a full scale test has been envisaged to verify the durability and damage requirements imposed to the structure.

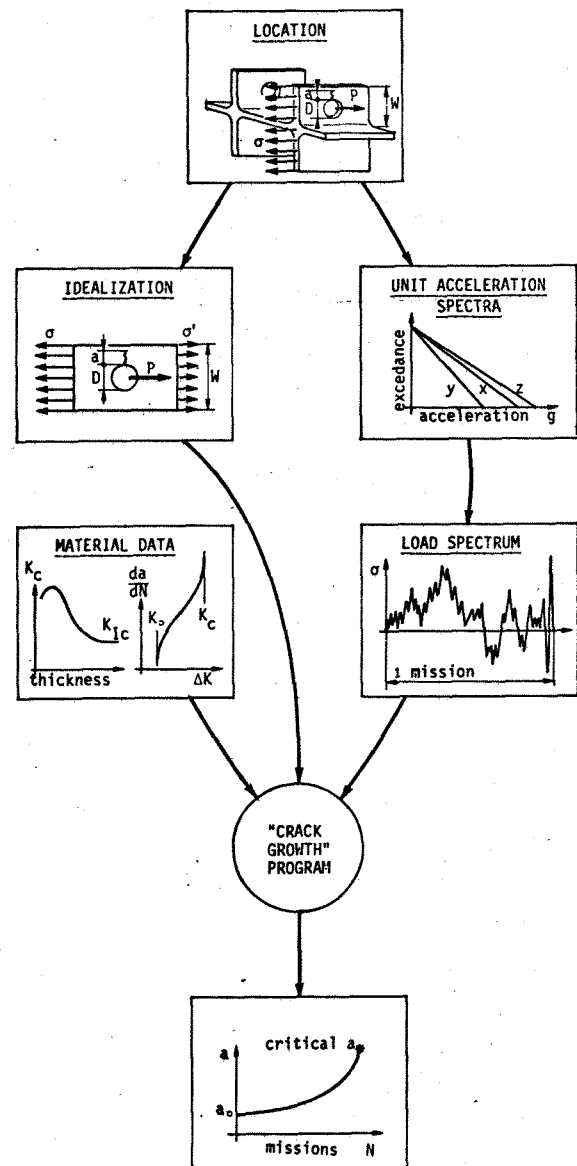
The purpose of such test is to evidentiate possible areas that have not been covered by analysis where the design could result to be critical for repeated loads, and to assess the ability of the structure to withstand the initial defects for the required life and eventually to define the inspection interval and methods of inspection to be adopted if the above requirements are only partially met.

The test will be run to cover the spectrum of loads resulting from the mission both pressure and inertia loads; two times the lifetimes (100 missions) will be utilized to assess the fatigue behaviour, then on the same article a limited number of defects of specified dimensions will be introduced in areas expected to be the most sensible to fracture propagation and two times the lifetime will be simulated checking at intervals the propagation of the cracks.

Extensive instrumentation will be used; strain gauges fatigue type and position transducers are utilized for the fatigue part of the test and crack wire gauges for the fracture mechanics part.

The test article will be periodically inspected and non-destructive controls, like X-rays liquid penetrant will be performed on selected areas to monitor the damage propagation.

The information obtained from the test will be compared with the analytical results and will constitute the final verification of the design of the Module Structure of the SPACELAB.



FLOW DIAGRAM OF CRACK PROPAGATION ANALYSIS

Figure 2

### 4. Research activities in Italy

To enhance the research in the field of Damage Tolerant Design in Italy a group was formed, contributed by the "Istituto di Ingegneria Aerospaziale of the Politecnico di Milano", the "Istituto di Aeronautica of University of Pisa", the "Istituto di Progetto di Aeromobili of the Politecnico di Torino" and the "Space Sector of AERITALIA S.p.A."

Such group has a fairly good potentiality, as it can rely on the research facilities of four different organizations and on the activity of about 25 researchers, whose experience covers a relatively wide range, from theoretical to technological

subjects.

The main financial support of the group so far came from the "Consiglio Nazionale delle Ricerche" (C.N.R.)<sup>(°)</sup>, responsible for the promotion of the Government supported Researches in Italy, with other connections, and minor supports from other Organizations.

The first research term covering 18 months, was completed within the end of October 1977. A second term is now on and a new term will start at the end of the second one <sup>(°°)</sup>.

The management of the research project was devolved to a research committee, composed by the Authors of this paper, which was responsible for the planning and the definition of the whole program, and for the evaluation of its results; this proved to be an effective means to coordinate the efforts of the different laboratories.

The first activity of the group was a review of the current state of the art, which allowed to locate some fields where the available knowledge was not adequate for a reliable design approach. Consequently a more detailed planning could be made.

An outline of the project plan is depicted in Fig. 3, together with its main flow lines.

The results of the whole plan are intended to be integrated with the available knowledge in the Manual "Fracture Mechanics Methods in the Design of Aerospace Structures".

Moreover work has been initiated on the definition of a computer program for the evaluation of the Safety and the Durability of aerospace Structures containing possible flaws or cracks, and subjected to variable loads.

The most relevant results obtained so far and not yet fully published are briefly presented and discussed in the next part of the paper.

Bibliographic quotations are also given at the end of the paper about other results already published.

#### 5. Numerical procedures for the evaluation of K in three-dimensional problems

The knowledge of the stress intensity factors along a crack front is necessary to evaluate the evolution of the crack front itself, by means of a suitable propagation law [1].

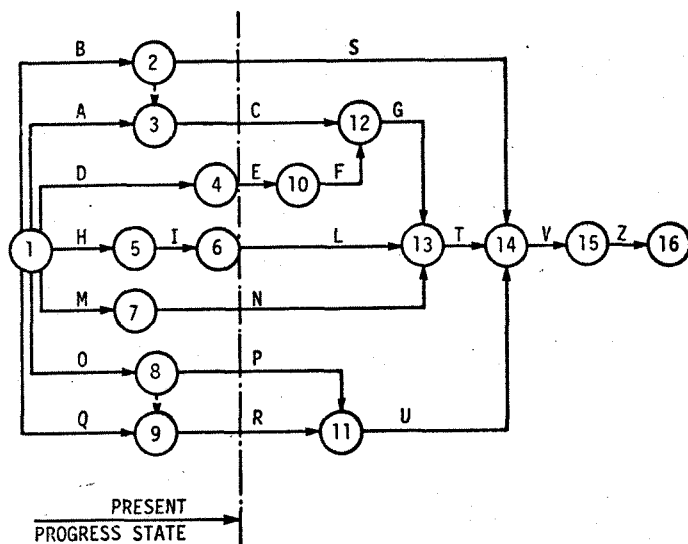
Stress intensity factors in closed form are available in the literature, for certain significant classes of geometries and stress conditions.

In other cases the values of K pertaining to some particular geometry and stress condition is known from experimental tests or from numerical analysis.

But unfortunately, in the design of an advanced aerospace vehicle, it occurs often to have to evaluate cracks for which not any solution is available. One example of these situations is a part through crack located in the fillet between the skin and a stiffener of an integrally machined cover panel. These are typical three-dimensional problems,

(°) C.N.R. financed the "Research on Fracture Mechanics of Pressurized Space Vehicles subjected to acoustic Fatigue" with the contracts n.SAS-760030-760031-760032 and 760040.

(°°) The three University components were previously supported by Italian C.N.R. in an experimental research on "Fatigue Phenomena and Aeronautical Structure behaviour under Acoustical Loads" (contracts n. 73.00166 and 74.00374).



OUTLINE OF THE RESEARCH PLAN

Figure 3

- A - EXPERIMENTAL MEASUREMENTS OF CRACK GROWTH FOR TYPICAL AEROSPACE MATERIALS WITH CONSTANT AND VARIABLE AMPLITUDE LOADS
- B - EXPERIMENTAL OBSERVATION OF CRACK GROWTH IN STIFFENED COMPONENTS UNDER VARIABLE LOADS
- C - EVALUATION OF THE RELIABILITY OF THE MODELS FOR CRACK GROWTH IN AEROSPACE STRUCTURES
- D - FINITE ELEMENTS PROCEDURES BASED ON LEFM TO EVALUATE K IN SIGNIFICANT GEOMETRIES AND STRESS FIELDS
- E - EXPERIMENTAL MEASUREMENT OF K ALONG THE CRACK FRONT IN SPECIFIED TEST CASES
- F - COLLECTION OF RESULTS ABOUT K ALONG DIFFERENT EVOLVING CRACK FRONTS, IN TYPICAL GEOMETRIES AND STRESS FIELDS
- G - SPECIFICATION OF RELIABLE PROCEDURES FOR THE EVOLUTION OF A CRACK FRONT IN TYPICAL GEOMETRIES AND STRESS FIELDS
- H - DEVELOPMENT AND EVALUATION OF COMPUTATIONAL PROCEDURES FOR THE DYNAMIC RESPONSE OF STRUCTURAL COMPONENTS UNDER RANDOM EXCITATION
- I - EXPERIMENTS ON DYNAMIC RESPONSE OF TYPICAL STRUCTURAL COMPONENTS WITH SINUSOIDAL AND RANDOM EXCITATION
- L - SPECIFICATION OF RELIABLE METHODS FOR THE EVALUATION OF DYNAMIC RESPONSE
- M - EVALUATION OF THE RELIABILITY OF DIFFERENT NDI TECHNIQUES
- N - SPECIFICATION OF THE SIZE OF INITIAL DEFECTS DEPENDING ON NDI METHODS AND STRUCTURE CHARACTERISTICS
- O - DEVELOPMENT OF NEW MODELS FOR DUCTILE FRACTURE TOUGHNESS
- P - EVALUATION OF THE RELIABILITY OF THE MODELS FOR EVALUATING RESIDUAL STRENGTH OF CRACKED AEROSPACE MATERIALS
- Q - EXPERIMENTAL MEASUREMENTS OF THE TOUGHNESS OF TYPICAL AEROSPACE MATERIALS, ALSO IN LARGE DUCTILITY SITUATIONS
- R - COLLECTION OF RELIABLE DATA ON FRACTURE TOUGHNESS OF AEROSPACE MATERIALS ALSO FOR LARGE DUCTILITY SITUATIONS
- S - DEVELOPMENT OF DESIGN PROCEDURES FOR MINIMUM WEIGHT STRUCTURAL ELEMENTS WITH SPECIFIED SAFE CRACK GROWTH LIFE
- T - SPECIFICATION OF RELIABLE PROCEDURES FOR EVALUATING THE SAFE CRACK GROWTH LIFE OF AEROSPACE STRUCTURES IN TYPICAL SITUATIONS
- U - SPECIFICATION OF RELIABLE PROCEDURES FOR EVALUATING THE RESIDUAL STRENGTH OF AEROSPACE STRUCTURES
- V - PRODUCTION OF THE MANUAL: FRACTURE MECHANICS METHODS IN THE DESIGN OF AEROSPACE STRUCTURES
- Z - DEVELOPMENT OF A COMPUTER PROGRAM FOR THE EVALUATION OF SAFETY AND DURABILITY OF AEROSPACE STRUCTURES CONTAINING POSSIBLE CRACKS AND FLAWS

where the knowledge of the values of  $K$  in several points of the crack front may be needed to evaluate the different propagation velocities of the same points, and hence the possible distortion of the crack front itself, which may be relevant to evaluate LBB ( $^{\circ}$ ) behaviour.

Numerical analysis based on "Finit Elements" might be a flexible tool for solving such problems, but usually it is not very efficient, as it is affected by large errors in  $K$  evaluation, unless a rather fine mesh is employed in the crack tip area. This will cause the size of the three-dimensional analyses to become so large that the cost and the time required may be unacceptable, especially when several shapes and sizes of the crack front must be analyzed to follow its evolution.

As it is well known, [36] improvements in the accuracy of  $K$  evaluation can be obtained by the use of elements containing singularities, but this will require a different mesh for every crack front, while it would be desirable to have an algorithm allowing the analysis of several crack fronts with the same mesh.

In order to obtain the latter feature an algorithm has been developed which allows to reduce or enlarge the crack size by simply activating or removing constraints between the nodes of adjacent elements.

Such constraints are obtained by means of the Lagrange's multipliers technique, which allows to compute the values of the stress intensity factors, pertaining to the different opening modes, performing basically only one major analysis [2]. With the same technique, plus some further development, also one-sided contacts may be simulated, which are likely to occur in the partial closure of a crack in bending. [3][4].

The above algorithm has been implemented in a general purpose FE program ( $^{\circ}$ ) [5], with  $K$  evaluation based on the well known crack closure Work Method and on the Interpolation Method.

As previously noted, the possibility of evaluating different crack fronts in a single basic analysis requires the use of non-singular elements.

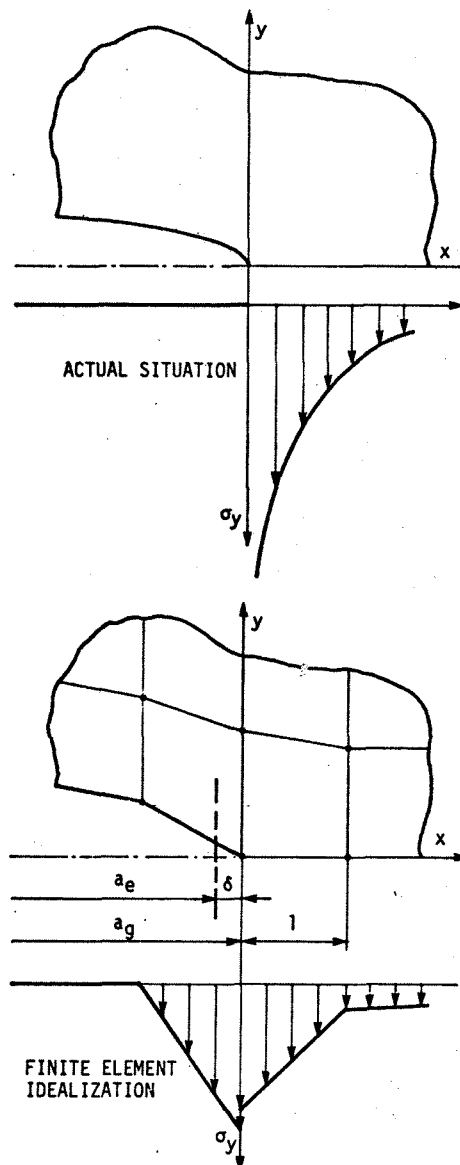
It has been observed that the relatively large systematic errors in  $K$  related to the mesh size can be strongly reduced, by simply considering an equivalent crack front, differing from the geometrical crack front of the FE idealization, by a displacement proportional to the mesh size itself. This corresponds to apply large corrections for the coarser meshes.

Several comparisons have been made with closed form solution, and with FE analysis using singular elements, for two-dimensional problems; in all the cases the equivalent crack front concept allowed to keep the errors in  $K$  below 1%, even with rather coarse idealizations [2]. The displacement of the equivalent crack tip respect to the geometrical crack tip was found to be a reduction in crack length, as it is obvious because the usual non-singular elements give a crack surface which is not stress-free near the tip, as in reality (Fig. 4).

Such reduction  $\delta$  in crack length has a constant value when the length of the elements along the crack is constant. For instance for rectangular linear elements (4 nodes)  $\delta = \ell/4$ , where  $\ell$  is element length; for parabolic elements (8 nodes) it depends

( $^{\circ}$ ) Leakage before burst.

( $^{\circ}$ ) The program FIESTA originally developed at the Istituto di Aeronautica of Politecnico di Milano [37].



CRACK SURFACE DEFLECTION AND STRESSES IN FINITE ELEMENT IDEALIZATION

Figure 4

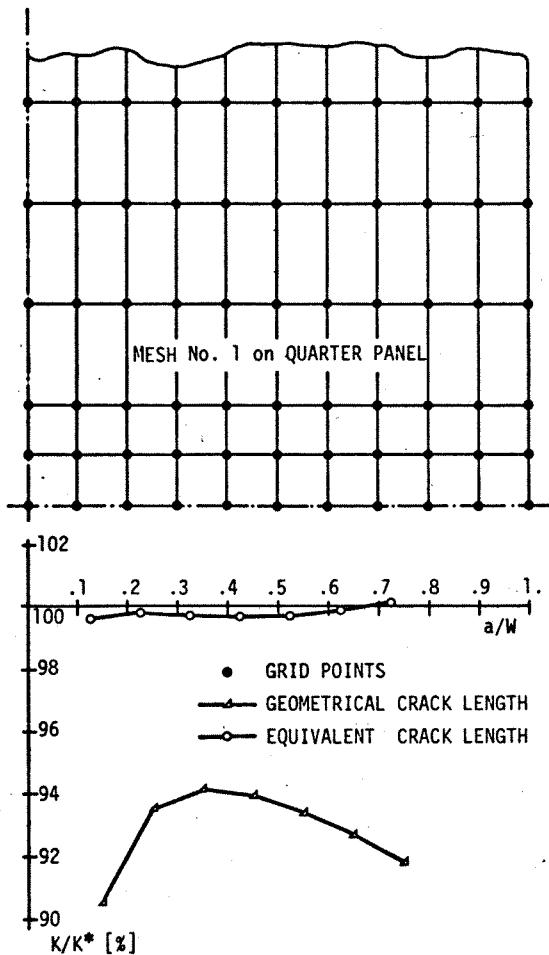
on the order of numerical integration, having the value  $\delta = \ell/10$  for the order 2.

As an example Fig. 5 shows the relative errors of the values  $K$  obtained in a single analysis, with the crack closure Method, for a center cracked panel, respect to the theoretical value [38].

$$K^* = \sigma \sqrt{\pi a} \sec \frac{\pi a}{w}$$

It may be seen that the equivalent crack tip concept reduces such errors well below 1%, even in the worst cases, for instance, when less than two elements are present in the half crack length.

The evaluations of three-dimensional cases have been rather limited, because reliable data are very scarce in the literature.



ERROR REDUCTION WITH EQUIVALENT CRACK LENGTH

Figure 5

The equivalent crack-front concept seems to work even in these situations, at least if the mesh is fairly regular on the crack surface, but more research is needed in this field [6].

Experimental measurements of  $K$  in suitably selected test cases have been planned, to obtain the data needed to evaluate the effectiveness of the algorithm and the reliability of the criteria developed.

The experience so far collected indicates that such FE procedures may become an efficient tool for producing the data on stress intensity factors needed to implement a reliable method for predicting the evolution of cracks and flaws in typical aerospace structures.

## 6. Models for ductile fracture

The relative ductility of the materials employed

and the smallness of the sheets thickness often cause the fracture behaviour of aerospace structures to be definitely ductile.

One of the most suitable criterion to evaluate the ultimate strength of a cracked element with an extensively ductile material behaviour seems to be the one based on the critical value of the Crack Tip Displacement (CTD) [7] (i.e. the measure of the opening of crack tip occurring at the onset of unstable propagation). Unfortunately the accurate measurement of such critical value is rather difficult, requiring long and delicate experimental techniques, so it is unlikely to become a common practice [39].

Another sound criterion is the one based on the critical value of some contour integral, as the well known  $J$ -integral originally demonstrated by Rice [40].

Generally such contour integrals have the limitations that they apply only to two-dimensional configurations, and their meaning as fracture toughness parameters holds only if the behaviour of the material can be considered non-linear elastic, that is to say as long as strain is increasing in every points.

But it is well known that during the stable propagation there is a definite region, a kind of wake traced by the moving tip area, where stress and strain are decreasing [41].

So the meaning of  $J$ -integral, and then its correlation with CTD, must cease when stable extension starts.

This appears to be a strong limitation in the use of  $J$  to specify ultimate fracture strength, as the stress corresponding to crack instability is often significantly higher than the stress at the onset of stable extension.

Nevertheless such integral has the advantage that it can be measured with a good accuracy. In the case of the simple geometry of a center cracked panel the contour integral can be evaluated simply from strain measurements taken along an appropriate contour [8][9].

An experimental technique for the above measure has been developed; it makes use of a limited number of strain gages, located in optimized positions. Fig. 6 shows the positions of strain gages used in the tests: strain gages were put only in one quarter in the center-cracked panel, on both panel surfaces, giving rise to 14 measuring bridges. This technique has the advantage of allowing a "direct" measure, i.e. a measure non depending on further assumption; an error analysis, carried out to evaluate the effect of errors in strain gage locations and in the measurement of bridges unbalance, estimated that, with the available technology, the final error in  $J$  evaluation could be below 2%.

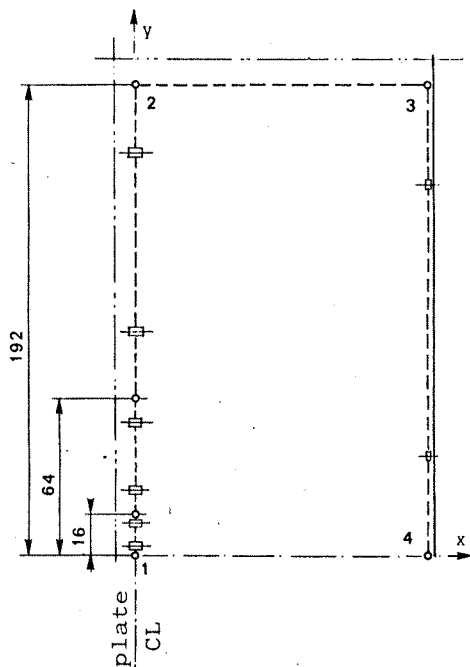
The experimental set-up is shown in Fig. 7; the use of a data acquisition system controlled by a small computer allowed to speed up the entire procedure, and to have the values of  $J$  plotted on line; the TV equipment was used to record stable crack extension, together with the measure of the gross area stress, the latter being a means to synchronize the position of the crack tip with the other recordings.

The results of the first tests, [8], are plotted in Fig. 8. It may be noted that they show a very smooth trend; moreover they were found to be in a very good agreement with results obtained with a more classical procedure [9][10].

A definite change in the trend of  $J$  was expected to appear at the onset of stable crack extension



(that was below  $\sigma = 25 \text{ kg mm}^{-2}$ , for all specimens), but actually this was not perceptible.



--- J contour

o extremities of the integration intervals

⊖ T rosette

⊕ single gage

#### EXPERIMENTAL EVALUATION OF J-INTEGRAL FROM STRAIN GAGE MEASUREMENT

Figure 6

A basic approach to the equilibrium of a ductile fracture [11][12] lead to the idea of a model consisting of a rigid perfectly plastic strip of vanishing thickness [38], lying in the crack surface, embedded in an elastic-plastic strain-hardening sheet [12].

This model is apt to reproduce the blunting of crack tip prior to stable extension, and hence to allow an easy evaluation of CTD as the elongation of the rigid-plastic strip at crack tip; moreover it can be analyzed with usual general purpose non-linear FE Programs.

Computations were carried out with the Program NONSAP [42], with different mesh sizes, keeping the crack tip in a fixed position, with monotonically increasing load histories.

From these analyses, besides CTD, also J-integral was evaluated, substantially along the same contour used in experiments [8].

A good correlation was found between CTD and J, as it was expected, since the analyses were done with fixed crack tip and increasing loads, and the contour integral kept its validity all the way.

Surprisingly, a good agreement was found between the values of J computed with such a model (with a fixed crack tip) and the experimental values obtained with the techniques outlined above, even in the stable propagation stage [8][13].

This agreement can be seen in Fig. 9, where com-

puted and experimental values are reported [13].

The reasons for this are not clear, and need further research; so far they do not seem to depend on some special mesh size, at least for the material 2024 T3 which was investigated.

Possibly contour integrals have some kind of meaning, at least with a certain approximation, also in the stable propagation phase.

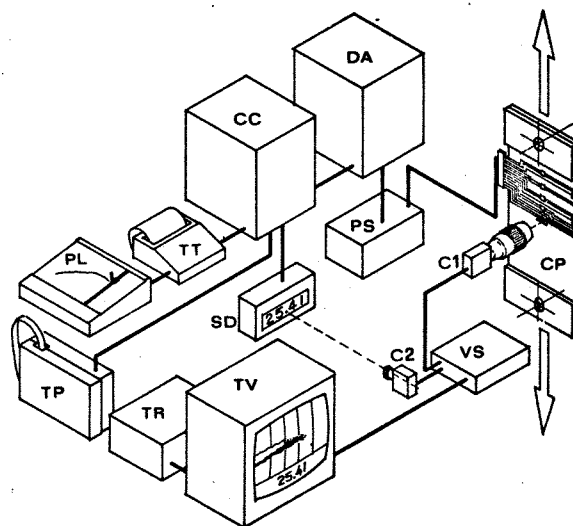
Besides its basic interest, this fact may have also practical importance; for instance if a correlation between CTD and J can be demonstrated up to crack instability, J measurements could be used to determine the toughness of ductile materials with center-cracked specimens, and the corresponding critical value of CTD to find crack instability with FE non-linear models.

#### 7. Fatigue crack propagation in riveted stiffened structures.

The main objective of this investigation is to produce an adequate set of crack growth data in riveted stiffened built-up structures to assess the predictability of the growth of a crack through the  $\Delta K$ -rate relationship of the material.

The need of such an investigation stems from the uncertainty which affects the evaluation of the stress intensity factor K in a cracked stiffened structure.

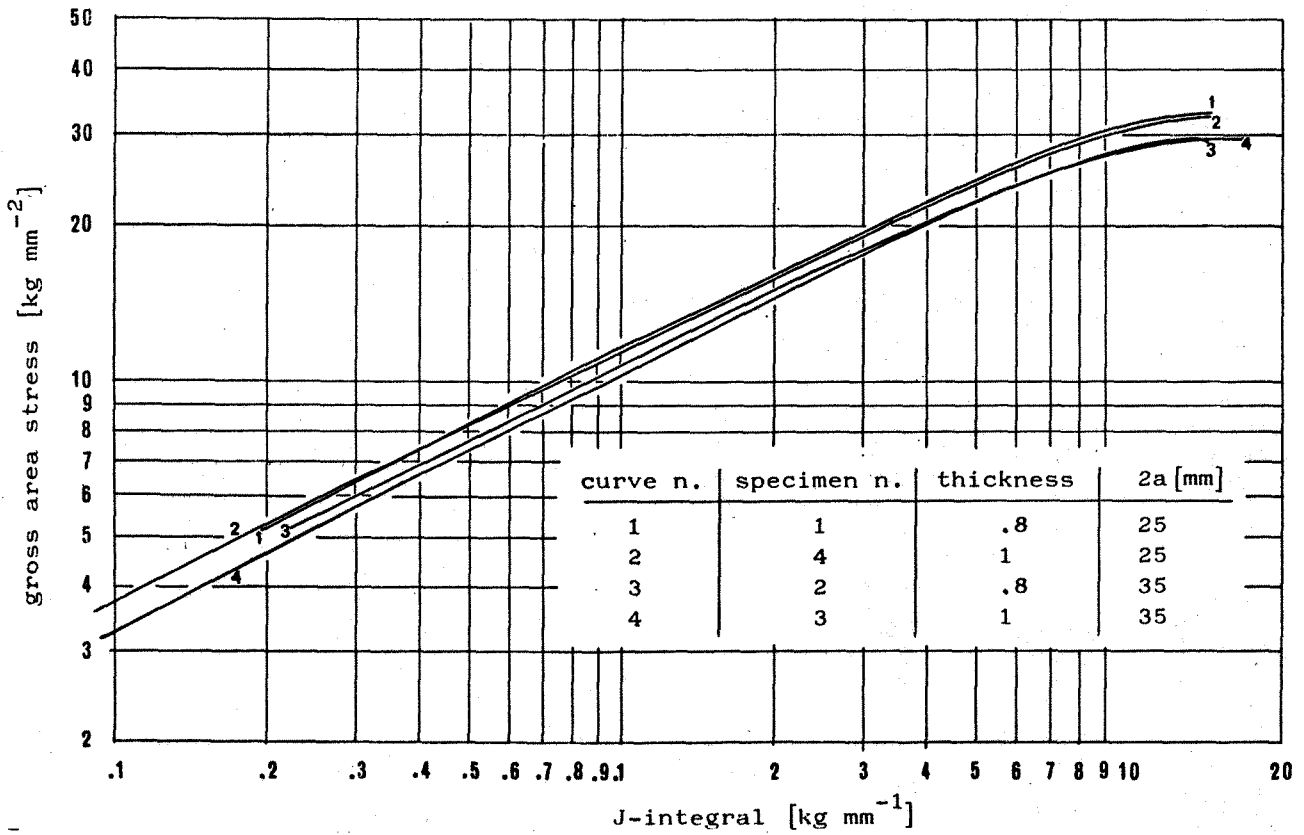
The stress intensity factor relative to a given crack length depends strongly on the mutual forces which take place in the joints between the stiffeners



EXPERIMENTAL INSTALLATION (schematic)

CC computer	SD stress display
CP cracked plate	TP tape punch
C1 TV camera (macro lens)	TR video tape rec.
C2 TV camera n.2	TT teleprinter
DA data acq. system	TV monitor
PL plotter	VS video signal mixer
PS strain gages power supply and 1/2 bridge	

Figure 7



PLOT OF EXPERIMENTAL RESULTS

Figure 8

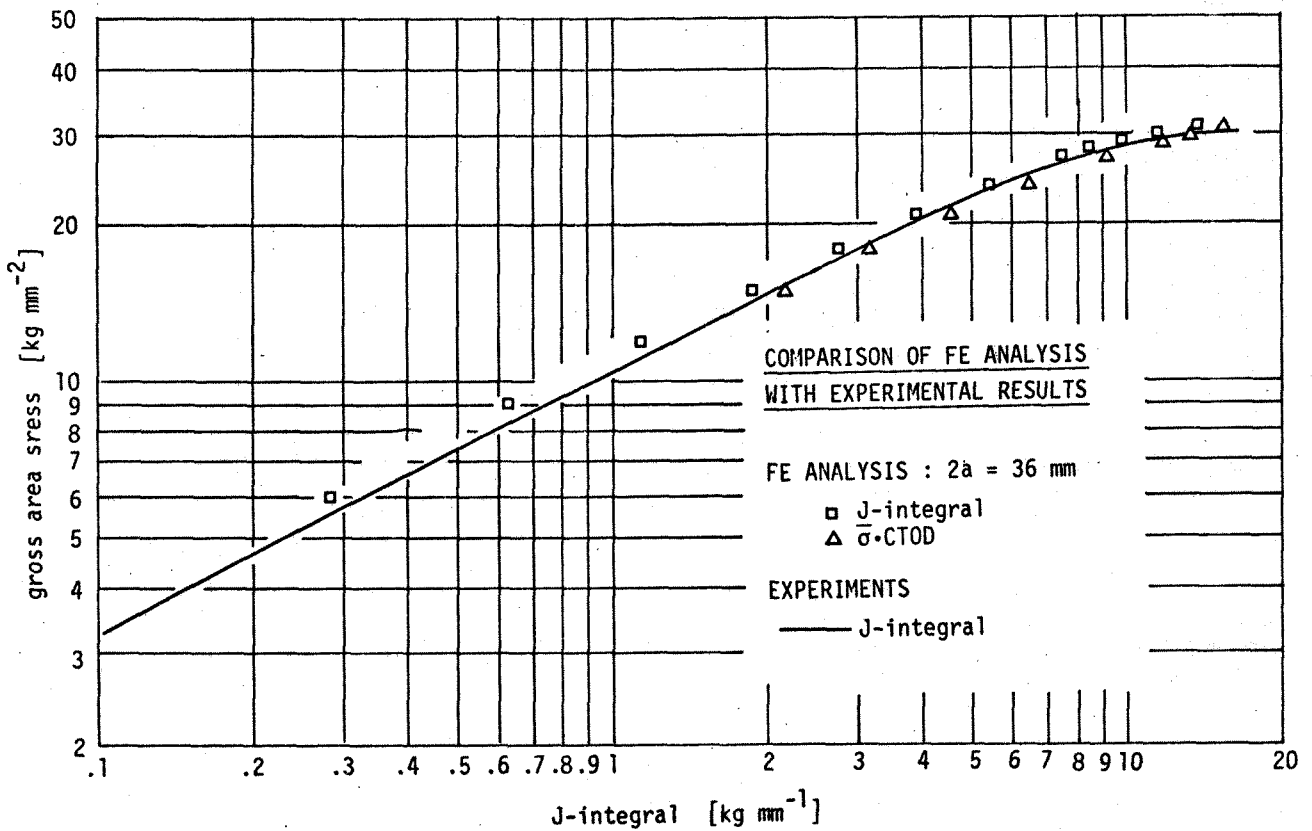


Figure 9

and the skin due to the load transfer from the cracked components to the integer ones. The accurate evaluation of such forces requires an efficient idealization of the deformations of both rivets and holes. As this idealization is intrinsically difficult the K values obtained with the existing computational approaches are frequently inaccurate.

The approach followed in pursuing the research objectives was, at first, the generation of fatigue crack growth data through tests of stiffened panels of different configuration (stiffener geometrical shape, rivet type, initial crack length and position). The results of such tests were processed using theoretical K value based on increasingly sophisticated models (rigid rivet, elastic flexible rivets, friction effects).

Through the comparison of the different K-rate relationships with the expected one (the one of the basic material) it was possible to reach a classification of the behaviour of the different stiffened panel configurations explaining at least qualitatively the role played by the joint in the crack growth under constant amplitude loading.

In the following the main results so far obtained [14], [15], [16], [17], [18], will be briefly summarized.

Fig. 10 shows the scatter band for each panel geometry, its position with the expected  $\Delta K$ - $da/dn$  curve of the material, and the representative shapes of the  $\Delta K$ -rate relationship in each band when K is computed on the basis of a rigid fastener approach. This figure summarizes the data from 52 crack propagation tests of stiffened and unstiffened panels.

The double-strip stiffened panels have a crack rate lower than predicted by the  $\Delta K$ -rate relationship of the material, while the opposite happens for all the other type of stiffened panels. The behaviour of the double-strip stiffened panels is most probably due to the friction forces between stiffener and sheet which help the rivets in conveying the load from the cracked sheet to the stiffeners.

The higher crack growth rate found with the other types of stiffened panels is probably due to the prevailing effect of the rivet flexibility.

The results shown in Fig. 11 demonstrate the inadequacy of computational approaches of K based on a rigid fastener hypothesis and display the complex effects of the joint geometry on the crack growth phenomenon.

Fig. 11 shows typical improvements of crack growth prediction in one-side-stiffened panels utilizing  $\Delta K$  values obtained by a flexible fastener approach. Fig. 12 shows similar improvements in the case of double strip stiffened panels obtained taking into account the friction effects in computing K.

The summarized results shows that an accurate prediction of the crack growth in a stiffened structure demands an adequate idealization of the joint behaviour in the computational methods of the stress intensity factor.

At present no idealization is available which works successfully in all the cases of technical interest. Further researches aimed at improving such a situation are in progress.

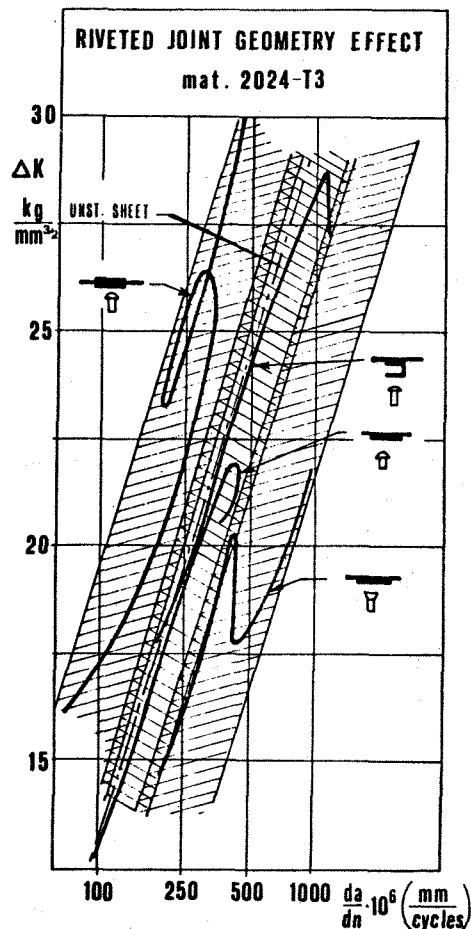


Figure 10

## 8. Reliability of crack growth computation methods

The purpose of this investigation is the development of a rationale to estimate the reliability of existing methods for predicting the growth of a crack under variable amplitude loading.

Such a prediction is a very complicated problem presently not fully understood. Existing methods largely rely on empiricism and derive their evidence from the ability to cope with an adequate set of test data. As the growth of a crack is a random phenomenon and the predictive methods are deterministic any comparison between measured and computed crack growth data must be judged on a statistical basis.

The logic of the present approach is shown in the Fig. 13. The detailed results of the investigation are given in ref. [20].

The constant amplitude line refers to the generation of test data relevant to constant amplitude loading and their statistical treatment. The constant amplitude line is essential since the prediction methods relevant to variable amplitude loading are typically cycle by cycle computation procedures which rely on constant amplitude  $\Delta K$ -rate relationship.

The constant amplitude line consists of several steps, namely:

- generation of experimental crack growth data by testing sheet specimens made of 2024-T3 aluminium alloys;
- generation of K-rate relationship in the forms given by Paris Forman and Collipriest laws through the computer program DADN. The DADN selects the constants which define such laws through a linear

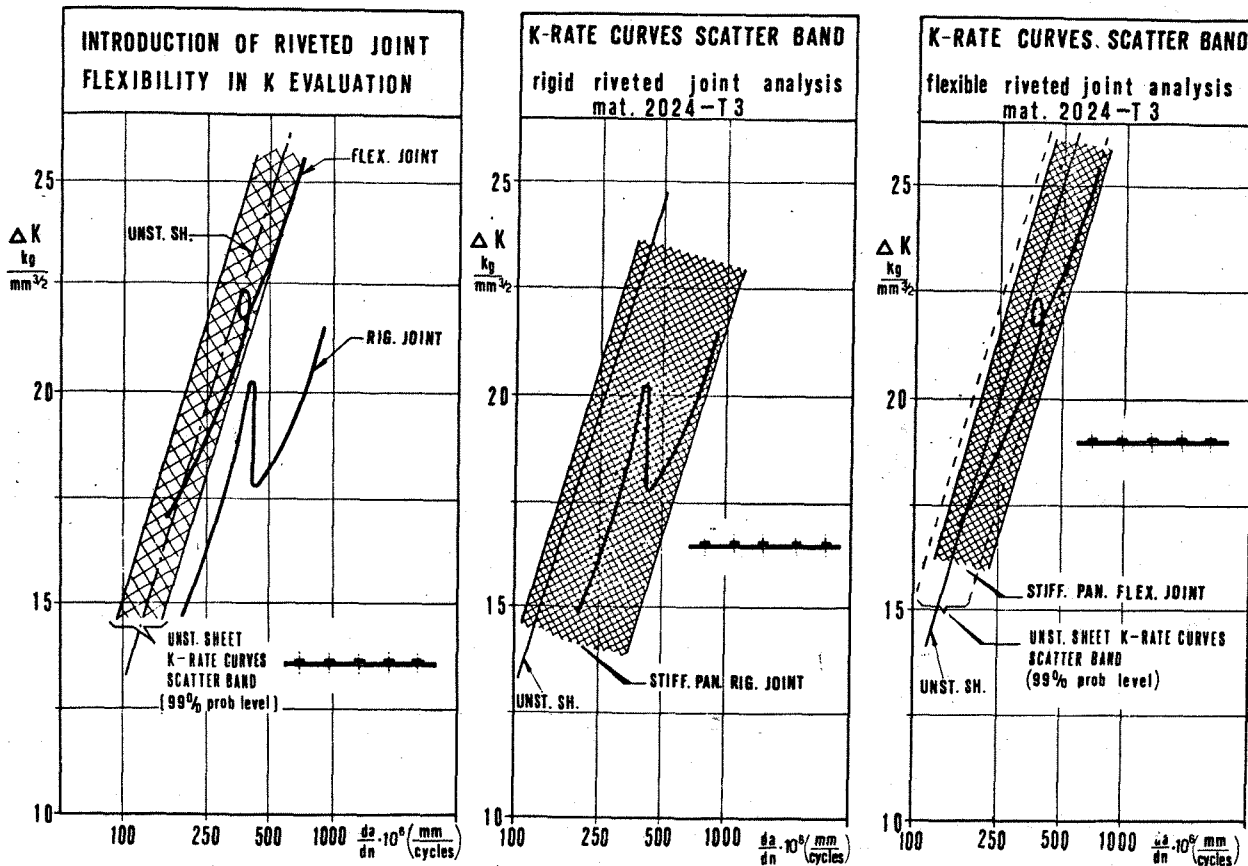


Figure 11

regression analysis substantiated by the usual set of significant tests;

- generation of the theoretical relationships between the crack length and the number of cycles of loading by integration of the best fit curves defined in the previous step;
- comparison of the experimental crack growth data with the theoretical ones. The comparison is obtained through the statistical treatment of the random variable  $N_s/N_c$ .

The evaluation of the reliability of variable amplitude crack growth prediction methods is based on the steps shown in the right hand side of figura 8.1, namely:

- variable amplitude crack growth tests conducted with a servohydraulic fatigue machine. Sheet specimens were used, drawn by the same sheet of constant amplitude specimens. Standardized spectrum like FALSTAFF and TWIST were used;
- prediction of the crack growth by the computer program CADAV [21] through the following methods linear Willemborg, Wheeler and Bell-Eidinoff. The constant amplitude crack propagation laws embodied in the method are the ones obtained in the constant amplitude line;
- comparison of the experimental crack growth data with the theoretical ones, through the statistical treatment of the random variable  $F_s/F_c$ .

Such a variable was found to conform to a Log-normal distribution. For each prediction method the mean value of the distribution furnishes a measure of the correctness of the method. In this way it is possible to compare on a rational basis the different approaches.

At present the variable amplitude line is still in progress since the variable amplitude tests are costly and time consuming.

The results until now obtained are limited and therefore no general conclusion can be drawn.

Nevertheless some trends are well established so that some comments can be tentatively tried.

The Wheeler method, with an adequate selection of the value of its plastic zone characteristic constant produces "exact" prediction, namely mean value of random variable  $F_s/F_c$  equal to zero. No definite results are currently available in the dependence on this constant from the type of spectrum.

The scatter of variable amplitude crack growth are generally larger than the same quantity under constant amplitude loading.

## 9. Minimum weight design of damage tolerant structures.

The objective of this investigation is the evaluation of the impact of damage tolerance requirements in the design of stiffened structures.

The problem was specialized to the design of the wing lower surface structure.

Several constraints must be allowed in the design of such structures. Some of these constraints refer to the damage tolerant characteristic, other to the static and fatigue strength requirements and other to the stiffness requirements. The principal constraints considered in the present investigation can be summarized as follows:

- a) the damaged structure must sustain a prescribed load for an assigned inspection interval;

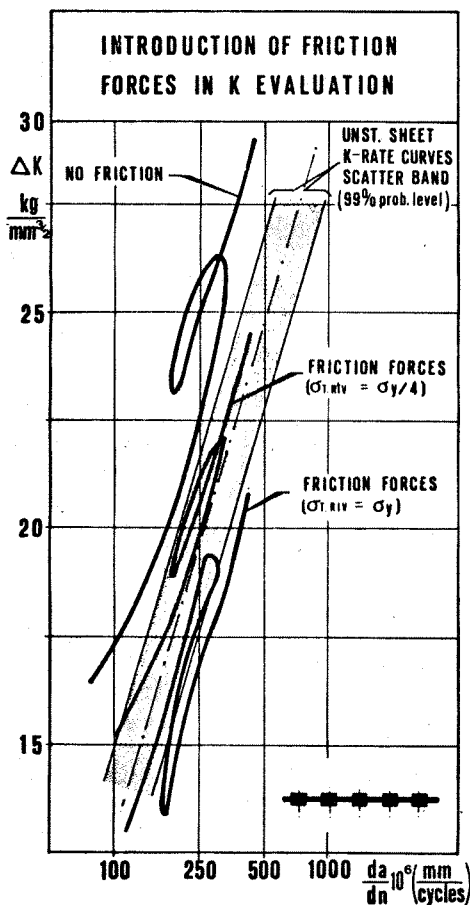


Figure 12

- b) an assigned damage (the largest crack or flaw missed in the last inspection) has to be allowed at the beginning on the inspection interval;
- c) the growth of the initial damage under a given load spectrum must be allowed in assessing the residual strength of the structure and its margin of safety in respect of the fail-safe load;
- d) the structure must sustain the compression loads (limit and ultimate) due to negative load factor flight conditions.

Compliance with the positive limit and ultimate loads requirements, aeroelastic requirements, crack free life requirements can give rise to further structural constraints.

The minimum weight approach was based on a modified penalty function methods.

Following such an approach the absolute minimum of the function.

$$w_j = w(x) + r_j \sum_{i=1}^n g_i^{-1}(x)$$

is searched through the Fletcher-Powell conjugate approach [22].

$W$  is weight of the structure,  $g_i$  are functions that represent the constraints previously discussed,  $X$  is the vector of the state variables which individuate the structure configuration;  $r_j$  is the  $j^{\text{th}}$  value of the penalty parameter.

As far as the constraints considered at the point d) are concerned, the  $g$  function is of the following type

$$g_i = 1 - \frac{\alpha_i N_x}{\bar{t} \cdot \sigma_b}$$

where  $N_x$  is the load per unit length corresponding to the one g flight condition;  $\alpha_i$  is the parameter chosen in such a way to obtain the load per unit length in the limit or ultimate load conditions;  $\bar{t}$  is the mean thickness of the panel;  $\sigma_b$  is a buckling or crippling stress.

The constraints stated at the points a), b), c) can be quantified in different way in connection with the structural concepts adopted to guarantee the damage tolerant characteristics.

If the slow crack growth concept is used the constraints stated in the previous points give rise to an unique  $g$  function of the type

$$g_i = 1 - \frac{\beta \cdot T}{E}$$

where  $\beta$  is an appropriate factor of safety,  $T$  is the inspection interval and  $E$  the endurance of the damaged structure; the functional relationship pertinent to such endurance is the following:

$$E = E(X, a_0, \Delta N_x, \alpha_{RS} N_x)$$

where  $a_0$  is the initial damage size  $\Delta N_x$  a scale factor specifying the assumed load spectrum,  $\alpha_{RS} N_x$  the prescribed maximum load that the damaged structure must sustain in the inspection interval.

The computation procedure for obtaining  $E$  is the following: for a given  $X$ , starting from the initial damage  $a_0$ , the growth of the crack in the sheet cover is evaluated, through a cycle by cycle integration procedure taking into account the stringer fatigue endurance through a method previously developed [18].

For each level of damage the residual strength of the structure is evaluated through the relationship  $K = K_c$ , and then the margin of safety with respect to  $\alpha_{RS} N_x$ . Finally the endurance  $E$  is obtained from the condition that this margin assume a prescribed value. This process to compute  $E$  is repeated for different  $X$  until the searched minimum of  $W_j$  is obtained.

The main results already obtained are reported in ref. [23] [24].

Fig. 14 shows the application of the described approach to the minimum weight design of the lower wing panel of a transport aircraft.

Fig. 15 shows the results obtained in terms of the mean thickness of the stiffened panel as function of the inspection interval.

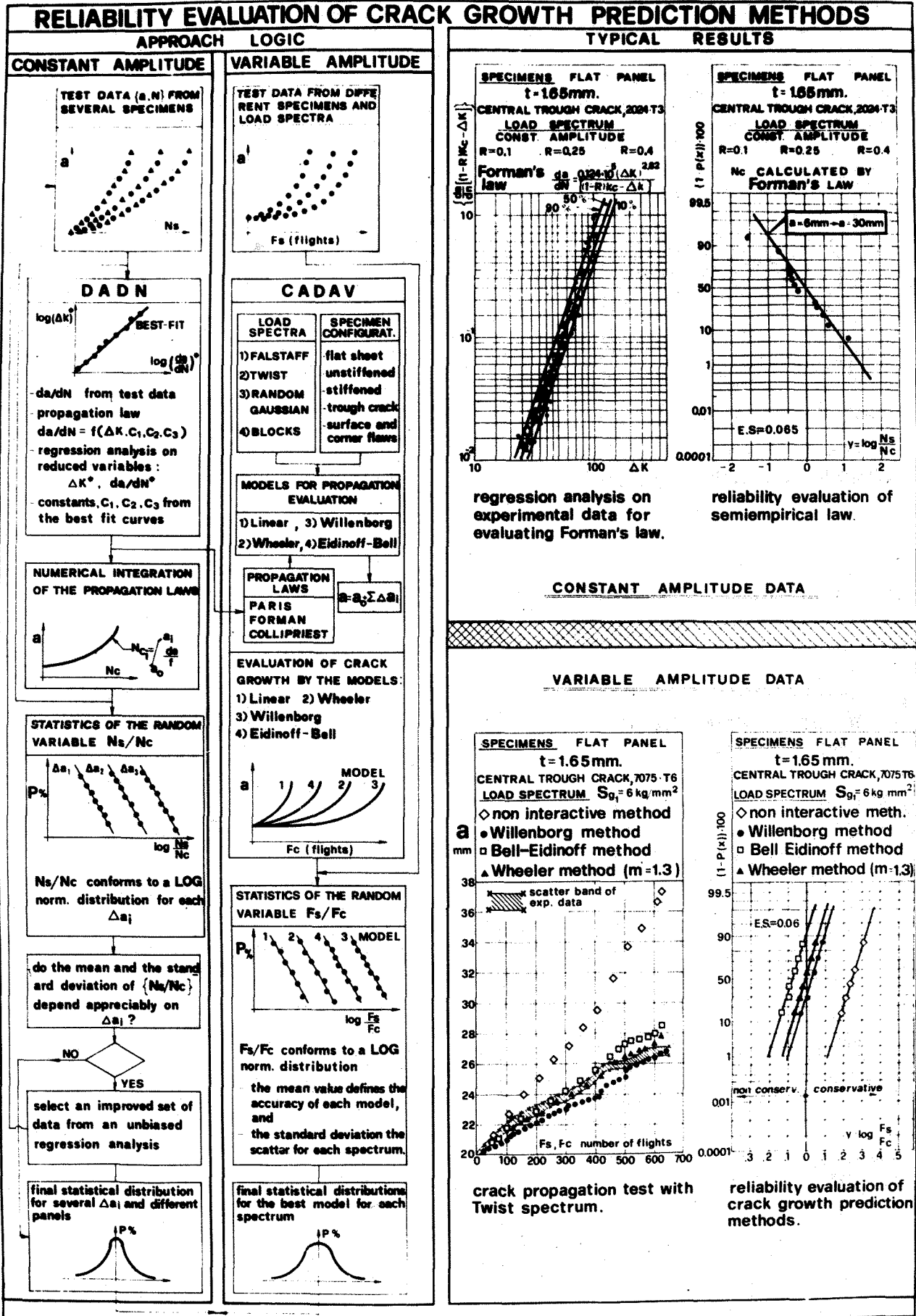
At present the minimum weight design of fail-safe crack arrest stiffened panels is being carried on. Together with the constraints previously discussed a further one is considered, namely that the structure must be able to operate for a specified period with a crack of specified length. The length at which arrest take place. In the current computation this length is assumed equal to two bays of the stiffened structure.

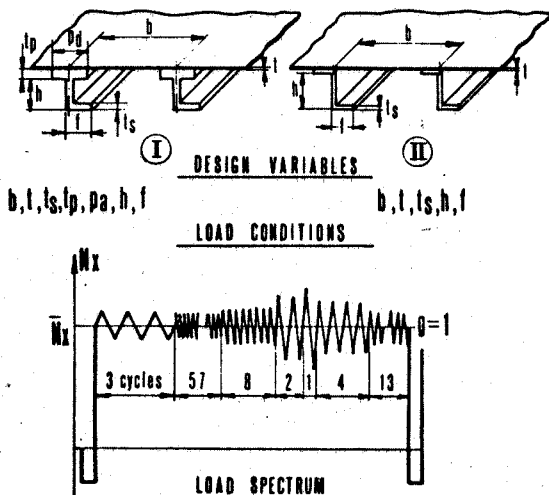
#### 10. Fracture strength of low gage aluminium alloy sheets.

The fracture behaviour of cracked low gage sheets is strongly influenced by plastic effects at the crack tips.

The current methods applied in evaluating the fracture strength of such elements generally embody one or more arbitrary hypotheses so that reliance on test data is necessary to substantiate their applicability.

The objective of this investigation was a comparative analysis of such methods on the basis of an omogeneous set of test data.





allowable load for damaged structure =  $(\alpha_{RS} \cdot N_x)$ ;  $\alpha_{RS} = 2.7$   
 limit load in compression =  $(\alpha_L \cdot N_x)$ ;  $\alpha_L = -1$   
 ultimate load in compression =  $(\alpha_U \cdot N_x)$ ;  $\alpha_U = 1.5$

Figure 14

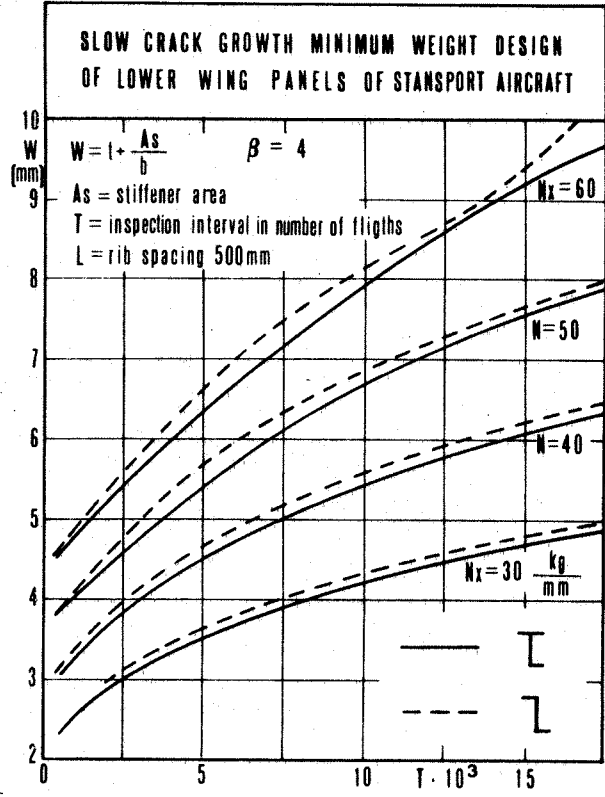


Figure 15

To this end a research program was carried out on the basis of the approach summarized in Fig. 16 where, together with the methods to be evaluated, the test procedure is schematically shown.

More in detail two test procedures were developed.

The applied load, crack length and panel elongation were measured simultaneously during the test. The applied load and the crack length were recorded by a camera, provided with a motor drive allowing 4 photograms per second and an optical equipment to assure a magnification high enough to clearly detect the crack tip; the applied load is revealed in the same photogram by an illuminated display close to the crack tips.

Then a single test can be elaborated according to all the methods except COD and J-integral.

The second test procedure was aimed at measuring J-Integral, and COD simultaneously together with all the other quantities. J-Integral is measured by a series of strain-gages in a suitable path on the panel, and COD by a sequence of couples of calibrated cantilevers so located that COD could be evaluated at the successive positions of the crack tip, during the slow crack growth.

The results of more than 60 tests have been analyzed in the course of the investigation, on sheets of different dimensions made of 2024-T3, 7075-T6 and 2219-T851, 52 tests were relevant to the first test procedure and the others the more complete second approach. The main results of the investigation are collected in [9], [10]. They can be summarized as follows:

- satisfactory methodology has been set up to carry out Fracture Mechanic tests; it is simple and quite reliable;
- the proposed methodology to measure COD and J Integral simultaneously seems simple and reliable even if only preliminary results were obtained;
- the obtained results can be utilized in the design of cracked plates loaded in tension;
- an omogeneous set of data has been generated which allows a sound comparison between the proposed methods.

### 11. NDI Methods

#### 11.1 Role of NDI

NDI must reliably provide detection of flaws and of their geometrical location and qualification.

A question may arise if a detected flaw is to be considered a defect, and thus unacceptable.

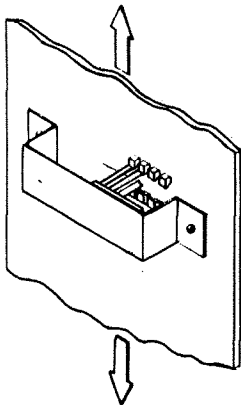
Detection and judgement are basically an uneasy and interdisciplinary commitment. Interpretation of NDI results should be delegated to a team of specialists rather than being left to the sole judgement of the NDI inspection. This team should be composed of a stress engineer, a material technologist, a laboratory expert and an NDI specialists, [27].

It may be said that almost every measurement of physical constants in metals has given rise to a particular NDI method. The five most commonly used methods for non-destructive evaluation of materials may be classified as follows:

- radiographic, magnetic particle, liquid penetrant, eddy current, ultrasonic.

Each of these methods shows in itself a great variety of modes of application and practically every method is available in a very diverse variety of hardware and software. Other techniques are currently undergoing development and many show promise as standard methods of the future. These include optical and acoustic holography, acoustic emission and

ANALYSIS OF FRACTURE MECHANIC METHODS.



- ANALYSIS AND REVIEW OF:
- 1) LINEAR ELASTIC FRACTURE MECHANICS.
  - 2) DUCTILE FRACTURE MECHANICS.
    - a) R-CURVE METHOD
    - b) EFTIS-LIEBOWITZ METHOD
    - c) FEDDERSEN METHOD
    - d) C.S.A. METHOD
    - e) BOCKRATH GLASSCO METHOD
    - f) COD METHOD
    - g) J-INTEGRAL METHOD

**COD AND J INTEGRAL TESTS.**  
 A SUITABLE EQUIPMENT WAS SET UP TO MEASURE COD AND INTEGRAL J WHEN CRACK LENGTH WAS GROWING.  
 COD IS MEASURED BY AN "AD HOC" EQUIPMENT AND J IS MEASURED SETTING STRAIN GAGES ALONG A SUITABLE PATH IN THE ELASTIC STRESSED ZONE

- CONSTRUCTION AND SET UP OF**
- 1) TEST EQUIPMENT APPARATUS
  - 2) MEASUREMENT METHODOLOGY TO DETERMINE:
    - a) PANEL ELONGATION  $\Delta l$  vs APPLIED LOAD P
    - b) CRACK LENGTH "a" vs APPLIED LOAD P

COD vs CRACK LENGTH "a",  
 J vs CRACK LENGTH "a",  
 COD AND J RELATIONSHIP

**FRACTURE MECHANIC TESTS**

- 1)  $\Delta l$  MEASURED BY A TRANSDUCER OF DISPLACEMENTS
- 2) "P" AND "a" ARE MEASURED SIMULTANEOUSLY BY A CAMERA; APPLIED LOAD IS VISUALIZED BY LIGHT DISPLAYS SET NEAR A CRACK TIP

3) MATERIALS: 2024-T3, 7075-T6 AND 2219-T851

- TEST EQUIPMENT FOR STATIC TESTS**
- STATIC TESTS**
- 1) RELATIONSHIP UP TO FAILURE
  - 2) STATIC CHARACTERISTICS OF MATERIALS

THE FRACTURE MECHANIC TESTS ARE ELABORATED ACCORDING TO ALL THE PROPOSED METHODS

<p><b>BOCKRATH-GLASSCO METHOD</b></p>	<p><b>C.S.A. METHOD</b></p> <p>DETERMINATION OF THE QUANTITY <math>C_m</math></p>	<p><b>EFTIS-LIEBOWITZ METHOD</b></p> <p>DETERMINATION OF <math>G_c</math> QUANTITY</p>	<p><b>FEDDERSEN METHOD</b></p> <p><math>K_c = \text{constant}</math></p>	<p><b>R-CURVE METHOD</b></p>	<p><b>LINEAR ELASTIC FRACTURE MECHANICS</b></p>
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Figure 16



thermal methods. Other methods, such as X-ray diffraction, will probably remain confined to laboratory uses.

With the development of new radio-sources nuclear techniques, once restricted to laboratory use, are becoming available for quality control in the field.

All methods have their own limitation because of their basic physics, mechanical arrangements, type of apparatus type of materials tested, accessibility to the point where a crack is starting or propagating and these limits are combined with human limitations of the testing personnel; consequently, the success of any NDI cannot be 100% reliable i.e. either all flaws are not detected or too many "flaws" are indicated which do not exist. In the first case the result is a technical risk, in the second the method may be uneconomical.

Therefore every organization has to build its own reliable detection limits in connection with the product, the operating condition.

### 11.2 NDI Capability statistical analysis.

To have a synthesis on the NDI method capabilities, one can refer to ad-hoc publications (see, for instance [27]).

A often used way of reporting data is the diagram of the "mean values or the confidence zones of the detection capability in function of the detection probability for given area interval and confidence level".

Another often used way is the diagram of the detection capability in function of the crack length (or depth for given probability and confidence level).

For each NDI method capability experimental tests are performed employing various operators. Each of them is requested to do observations, with the NDI methods subjected to evaluation on  $g$  groups of  $N_p > n_p$  specimens ( $p=1,2,\dots,g$ ),  $n_p$  of which contain an artificially created crack of given geometrical characteristics (for instance length  $l_p$ ).

The tests are based on the assumption that to each  $l_p$  and in the particular whole of test conditions there exists a defined probability  $p_p$  of detecting the crack in cracked specimens. The aim of the tests is the statistical determination of such  $p_p$  on a sample, eventually distinguishing between the various operators.

Since the  $u_p$  cracked specimens belong to a group of  $N_p$  specimens, there are the following possibilities:

- 1) cracked specimen which is detected
- 2) non cracked specimen which is not detected
- 3) cracked specimen which is not detected
- 4) non cracked specimen which is detected.

The current interpretations of experimental results take into account the results of the types 1) and 3), which are concerned with  $u_p$  specimens, to which they apply the theoretical results on the "repeated tests".

A possible way of thinking in order to have the statistical evaluation of  $p_p$  on a set of  $n_p$  specimens, would be the following commonly reported in elementary works of statistic.

The repeated tests follow the binomial distribution (stochastic variable  $k$ , i.e. the number of detected cracks)

$$P_R = \binom{n}{R} p^k (1-p)^{n-k}$$

For  $u \rightarrow \infty$ , by means of the Sterling-De Moivre formula, putting  $x$  as stochastic variable instead of  $k$ , the distribution approaches asymptotically the Gauss one:

$$P_x = \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\sigma_x}} e^{-\frac{1}{2}z^2} \quad (z = \frac{x-n}{\sigma_x})$$

If  $n$  is great it is possible to perform on  $p$  an analysis of the inferior limit of unilateral confidence interval corresponding to a given confidence level  $1-\alpha$ , on the base of the normal distribution.

In fact  $u = \frac{\frac{x}{n} - p}{\frac{\sigma_x}{n} \sqrt{n}}$  has a normal distribution.

Using the relation  $\sigma_x = \sqrt{np(1-p)}$ , valid in the binomial distribution, if

$$P(u_{2\alpha} < u < u_{2\alpha}) = 1 - 2\alpha,$$

$$P\left\{\left(\frac{x}{n} - p\right)^2 \leq u_{2\alpha}^2 \frac{p(1-p)}{n}\right\} = 1 - 2\alpha$$

$$P\left\{p \geq \frac{1}{2(n + u_{2\alpha}^2)} (2x + u_{2\alpha}^2 - u_{2\alpha} \sqrt{u_{2\alpha}^2 + 4X(1 - \frac{X}{n})}\right\} = 1 - \alpha$$

When it is necessary to elaborate also experimental results of not so great values of  $n$ , a procedure can be used that performs a stochastic evaluation of  $\sigma_x$  on the base of the admission that

$\eta = \frac{(n-1)x}{\sigma_x}$  has a  $\chi^2$  distribution of  $n-1$  degree of

freedom. Obtained the proper limit of unilateral confidence limit of  $\sigma_x$  for a given confidence level  $1-\alpha$  on inferior bound of unilateral confidence interval of  $n_p$  for a given confidence level,  $1-\beta$  is obtained from the Gauss's distribution, (for a criticism of such procedure see for instance [26]).

In fact the most part of the NDI method capability data are indicated as: probability of detecting the cracks for given probability (stochastic analysis of  $n_p$ ) and for a given confidence level (stochastic analysis of  $\sigma_x$ ) (see for instance [27], Phase I).

A straightforward correct procedure (which was used in ref. [27], Phase II) can be adopted with the aide of a modern computers. The binomial distribution of  $k$  can be used in order to obtain directly the inferior limit of unilateral confidence interval of  $p$  corresponding to a given confidence level.

This way of thinking seems to be very promising for further steps in the insight of NDI method capability and particularly in the theory of approaching safety as a whole probabilistic problem.

## 12. Concluding remarks

The development of effective methods for the design of Damage Tolerant Structures requires a deeper insight into Fracture Mechanics and NDI techniques.

In particular the evaluation of initial defects demands a sensible effort for the integration of different specialists. A good statistical model is also needed for the evaluation of the capability of NDI methods; such model will enable both a better planning of relevant experiment and a more rational use of their results.

As far as Fracture Mechanics is concerned the basic topics are the prediction of the growth of a crack under variable amplitude loads and the evaluation of its critical dimensions.

Since all the crack growth models assume the stress intensity factor as the basic fracture parameter, more efficient procedures are needed to evaluate  $K$  in all those situations arising from practice where known solutions cannot be used.

Another urgent need is a sound assesment of the reliability of the crack growth models themselves, particularly in the case of variable amplitude loading.

In the prediction of critical defect dimensions ductility may play an important role, demanding both the improvement of fracture models and the development of more suitable experimental techniques.

The joint research referred in this paper has pointed out and partially answered some of such questions; besides an example of the impact of Damage Tolerance requirements on structural design is also given, through the use of minimum weight approach.

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